

# C.U.SHAH UNIVERSITY

## Summer Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT3

Branch: Diploma (All)

Semester : 2

Date : 25/04/2018

Time : 10:30 To 01:30

Marks : 70

Instructions:

- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1

Attempt the following questions:

(14)

- a)  $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = \underline{\hspace{2cm}}$   
 (A)  $ax^{n-1}$  (B)  $nx^{n-1}$  (C)  $na^{n-1}$  (D) None of these
- b)  $\lim_{x \rightarrow 0} \frac{\sin 3x}{2x} = \underline{\hspace{2cm}}$   
 (A)  $3/2$  (B)  $2/3$  (C)  $1/3$  (D)  $1/2$
- c)  $\lim_{x \rightarrow 0} \left(1 - \frac{2x}{3}\right)^{\frac{4}{x}} = \underline{\hspace{2cm}}$   
 (A)  $e$  (B)  $e^{-8/3}$  (C)  $e^{2/3}$  (D)  $e^{-3/2}$
- d)  $\frac{d(\sec x)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $\sec x \tan x$  (B)  $-\sec x \tan x$  (C)  $\cos e x \cot x$  (D)  $-\cos e x \cot x$
- e)  $\frac{d(x^2 + 2x + 7)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $2x$  (B)  $2x + 1$  (C)  $2x + 2$  (D) None of these
- f)  $\frac{d(\tan^{-1} x)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $\frac{-1}{\sqrt{1-x^2}}$  (B)  $\frac{1}{\sqrt{1-x^2}}$  (C)  $\frac{1}{1+x^2}$  (D)  $\frac{-1}{1+x^2}$
- g)  $\frac{d\left(\frac{1}{x}\right)}{dx} = \underline{\hspace{2cm}}$   
 (A)  $\log x$  (B)  $-\frac{1}{x}$  (C)  $\frac{1}{x^2}$  (D)  $-\frac{1}{x^2}$
- h)  $\int e^x dx = \underline{\hspace{2cm}}$



(A)  $\log x + c$  (B)  $e^x + c$  (C) 1 (D) 0

i)  $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \underline{\hspace{2cm}}$

(A)  $\log \left| x + \sqrt{x^2 + a^2} \right| + c$  (B)  $\log \left| x + \sqrt{x^2 - a^2} \right| + c$  (C)  $\frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$

(D)  $\frac{1}{2a} \log \left| \frac{a+x}{a-x} \right| + c$

j)  $\int_0^1 \frac{2}{1+x^2} dx = \underline{\hspace{2cm}}$

(A)  $\pi$  (B)  $\frac{\pi}{4}$  (C)  $\frac{\pi}{2}$  (D) None of these

k)  $-3(5, -1, -2) + 8(1, 1, 0) = \underline{\hspace{2cm}}$

(A) (7, 11, 6) (B) (7, -11, 6) (C) (7, 11, -6) (D) (-7, 11, 6)

l) If  $\theta$  is the angle between the vectors  $\vec{x}$  and  $\vec{y}$  then  $\cos \theta = \underline{\hspace{2cm}}$ .

(A)  $\frac{|\vec{x} \times \vec{y}|}{|\vec{x}||\vec{y}|}$  (B)  $\frac{\vec{x} \times \vec{y}}{|\vec{x}||\vec{y}|}$  (C)  $\frac{\vec{x} \cdot \vec{y}}{|\vec{x}||\vec{y}|}$  (D)  $\frac{\vec{x} \times \vec{y}}{|\vec{x} \times \vec{y}|}$

m) If  $x(3, 1) + y(4, 2) = (1, 0)$  then  $x = \underline{\hspace{2cm}}$  and  $y = \underline{\hspace{2cm}}$ .

(A) 1, 2 (B) 2, 1 (C) 1,  $\frac{1}{2}$  (D) 1,  $-\frac{1}{2}$

n)  $(1, 1, 1) \cdot (-1, 2, -3) = \underline{\hspace{2cm}}$ .

(A) -2 (B) 2 (C) 1 (D) -1

**Attempt any four questions from Q-2 to Q-8**

**Q-2 Attempt all questions (14)**

a) If  $\vec{a} = 2i + j - k$ ,  $\vec{b} = i - j + 2k$  and  $\vec{c} = i - 2j + k$  then find the direction cosines of  $\vec{a} + \vec{b} - 2\vec{c}$ . (5)

b) Prove that angle between two vectors  $i + j - k$  and  $2i - 2j + k$  is  $\sin^{-1} \left( \frac{\sqrt{26}}{3\sqrt{3}} \right)$ . (5)

c) Evaluate:  $\lim_{x \rightarrow 2} \frac{x^7 - 128}{x^4 - 16}$  (4)

**Q-3 Attempt all questions (14)**

a) Prove that  $\lim_{x \rightarrow 3} \frac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \frac{\sqrt{35}}{5}$ . (5)

b) Evaluate:  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{2 - \sec^2 x}{1 - \tan x}$  (5)

c) If  $y = \frac{\log x}{x}$  then find  $\frac{dy}{dx}$  at  $x = 1$ . (4)

**Q-4 Attempt all questions (14)**



a)  $S = t^3 - 6t^2 + 8t - 4$  gives the distance travelled by a body in  $t$  seconds. Find velocity and acceleration at  $t = 4$  second. (5)

b) Find  $\frac{dy}{dx}$  if  $y = \log \sqrt{\frac{a+x}{a-x}}$ . (5)

c) Simplify:  $(10i + 2j + 3k) \cdot [(i - 2j + 2k) \times (3i - 2j - 2k)]$ . (4)

**Q-5**

**Attempt all questions**

(14)

a) Find unit vector which is perpendicular to  $\vec{a} = i + j + k$  and  $\vec{b} = 2i - 2j + k$ . (5)

b) Find  $\frac{dy}{dx}$  if  $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$ . (5)

c) Evaluate:  $\lim_{x \rightarrow 0} \frac{3^{2x} - 2^{2x}}{x}$  (4)

**Q-6**

**Attempt all questions**

(14)

a) Forces  $(1, 2, 3)$ ,  $(-1, 2, 3)$  and  $(-1, 2, -3)$  act on a particles and the particle moves from the point  $(0, 1, -2)$  to  $(-1, 3, 2)$ . Find the work done by the forces. (5)

b) Prove that  $\int_0^{\frac{\pi}{2}} \frac{\tan x}{\tan x + \cot x} dx = \frac{\pi}{4}$ . (5)

c) Find derivative of  $f(x) = e^x$  using definition. (4)

**Q-7**

**Attempt all questions**

(14)

a) Evaluate:  $\int x \log x dx$  (5)

b) Find the area of the region bounded between curve  $y = x^2$  and straight – line  $x = 2$ . (5)

c) For what value of  $R$ , vectors  $2i - 3j + 5k$  and  $Ri - 6j - 8k$  are perpendicular to each other? (4)

**Q-8**

**Attempt all questions**

(14)

a) If  $f'(x) = 4x^2 + 6x - 3$  and  $f(1) = 2$  then find function  $f(x)$ . (5)

b) Find  $\frac{dy}{dx}$  if  $y = (\sin x)^x$ . (5)

c) Evaluate:  $\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$  (4)

